Modelling Competition in `comsimitv` package

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Contents

During simulations seed production depends on the competition for resources within (sub)-communities. First strength of competition (α) is calculated for each pair of co-occurring individuals from the trait values related to resource use by competition kernels specified in `competition.kernel` parameter of `comm.simul` function. Then the matrix of pairwise competition coefficient are used in function specified by `fSeedProduction` parameter. This vignette shows the available symmetric (where $\alpha_{ij} = \alpha_{ji}$) and asymmetric (where $\alpha_{ij} \neq \alpha_{ji}$ if $i \neq j$) kernels, and `SeedProduction` function that recently the only available function in the package for this purpose.

1 Symmetric competition kernels

Recently the only available symmetric competition kernel is the Gaussian one:

$$\alpha_{ij} = \exp\left(-\frac{(B_i - B_j)^2}{\sigma_b}\right) \tag{1}$$

where B_i and B_j are the resource use related trait values of the two species, while σ_b determines how steeply decrease the strength of competition with increasing difference in resource use (Figure 1).

For $B_i = B_j$, $\alpha_{ij} = 1$ irrespectively to value of σ_b. If $\sigma_b = 0$ the strength of competition is zero any case of $B_i \neq B_j$. If $\sigma_b = \infty$, $\alpha_{ij} = 1$ for all species pairs.

Gaussian competition kernel can be used by setting `competition.kernel="Gaussian.competition,kernel"` (which is the default value of this parameter). Value of σ_b has to be set by parameter `sigma.b`.

According to MacArthur & Levins (1967), this competition kernel can be deduced as overlap of Gaussian resource use curves. Their general formula for overlap is

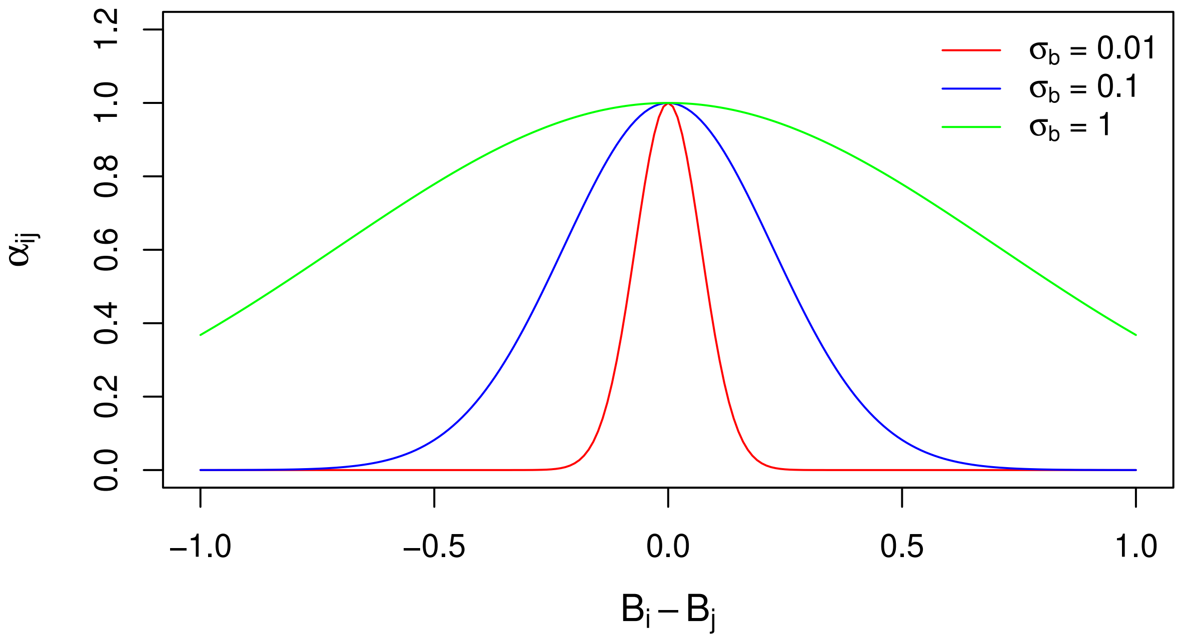

Figure 1: Shape of Gaussian competirion kernel with different σ_b values

$$\alpha_{ij} = \frac{\int_{-\infty}^{\infty} U_i(x) U_j(x)\ dx}{\int_{-\infty}^{\infty} U_i^2(x)\ dx} \tag{2}$$

where U is the resource use function, and x is the quality of the resource (e.g. seed size or rooting depth). Let both U_i and U_j be density function of normal (Gaussian) distribution with same standard deviation (σ), and let rescale x to be expected values equals to zero and $d = B_i - B_j$, respectively:

$$\alpha_{ij} = \frac{\int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{\sigma^2}\right) \exp\left(-\frac{(x-d)^2}{\sigma^2}\right)\ dx}{\int_{-\infty}^{\infty} \left[\exp\left(-\frac{x^2}{\sigma^2}\right)\right]^2\ dx} = \frac{\int_{-\infty}^{\infty} \exp\left(-\frac{x^2+(x-d)^2}{\sigma^2}\right)\ dx}{\int_{-\infty}^{\infty} \exp\left(-\frac{2x^2}{\sigma^2}\right)\ dx} \tag{3}$$

Since $x^2 + (x-d)^2 = 2x^2 + d^2 - 2xd = 2\left(x - \frac{d}{2}\right)^2 + \frac{d^2}{2}$

$$\alpha_{ij} = \frac{\exp\left(\frac{d^2}{2\sigma^2}\right) \int_{-\infty}^{\infty} \exp\left(-\frac{2(x-d/2)^2}{\sigma^2}\right)\ dx}{\int_{-\infty}^{\infty} \exp\left(-\frac{2x^2}{\sigma^2}\right)\ dx} = \exp\left(\frac{d^2}{2\sigma^2}\right) \tag{4}$$

Note that σ_b in equation (1) equals to $2\sigma^2$ in equation (4)

2 Asymmetric competition kernels

Recently two types of asymmetric competition kernels are available via `asymmetric.competition.kernel` function:

- Kisdi's convex-concave function
- smooth function suggested by Nattrass et al. (2012)

2.1 Kisdi's convex-concave function

It is a function defined by equation (2) in Kisdi (1999), however the parametrization are slightly modyfied:

$$\alpha_{ij} = C\left(1 - \frac{1}{1 + v\exp\left(-\frac{B_i - B_j}{\sigma_b}\right)}\right) \quad (5)$$

Contrary to the Gaussian competition kernel, It has three parameters (C, v, σ_b) instead of the only one parameter of Gaussian competition kernel. Note that in the R function these parameters are called `ac.C`, `ac.v` and `sigma.b`, respectively. C and v have to be positive, while $\sigma_b \neq 0$. Possible values of the function ranges from zero to C. If $\sigma_b > 0$ it is a decreasing sigmoid (convex-concave) function (Figure 2) of trait difference ($B_i - B_j$) with inflection point at $B_i - B_j = \sigma_b \ln v$, where the strength of competition is $C/2$.

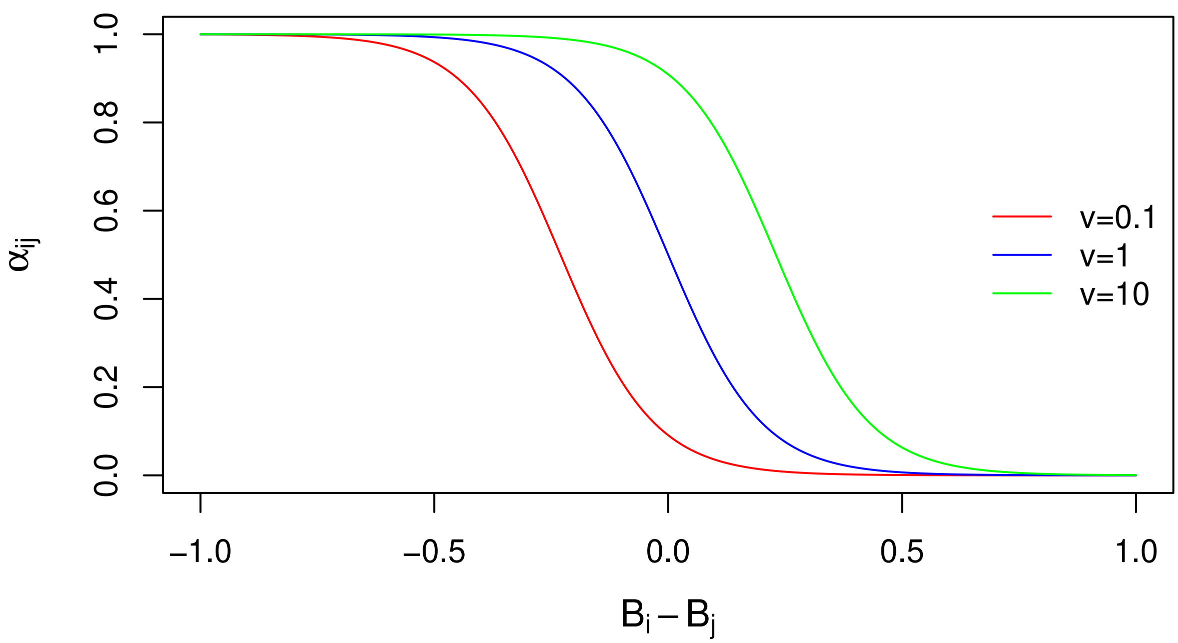

Figure 2: Shape of Kisdi's convex-convave function with different values of v (C=1,σ_b=0.1)

Strength of competition between functionally equivalent individulas (i.e. if $B_i = B_j$) is $\alpha_{ij} = C\left(\frac{v}{1+v}\right)$. If the other two parameter fixed, absolute value of parameter σ_b determines the steepness of the curve around its inflection point (Figure 3).

2.2 Smooth function poropsed by Nattrass et al. (2012)

This is also a sigmoid function defined by a formula similar to Kisdi's function:

$$\alpha_{ij} = 1 + C - \frac{2C}{1 + \exp(-\frac{2(B_i - B_j)}{\sigma_b})} \quad (6)$$

It ranges from $1 - C$ to $1 + C$. Position of its inflection point is $B_i - B_j = 0$, where its value is 1, irrespective to the parameter values. When its range (i.e. value of parameter C) is fixed, σ_b determines the steepness of the curve at the inflection point: lower σ_b results in steeper curve.

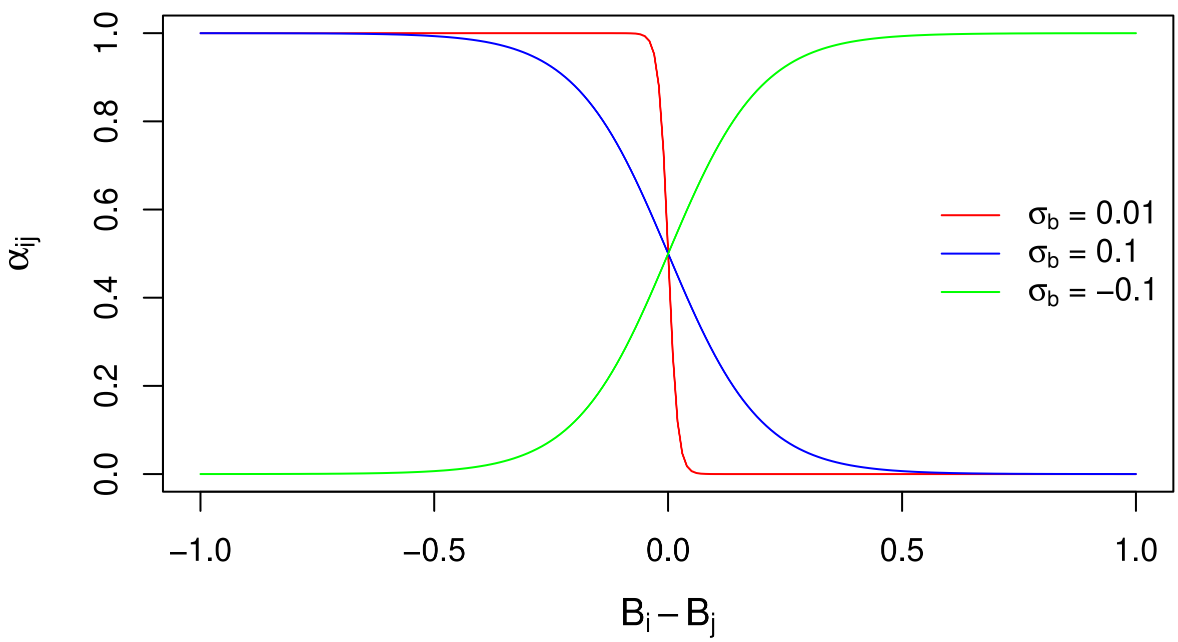

Figure 3: Shape of Kisdi's convex-convave function with different values of σ_b (C=1,v=1)

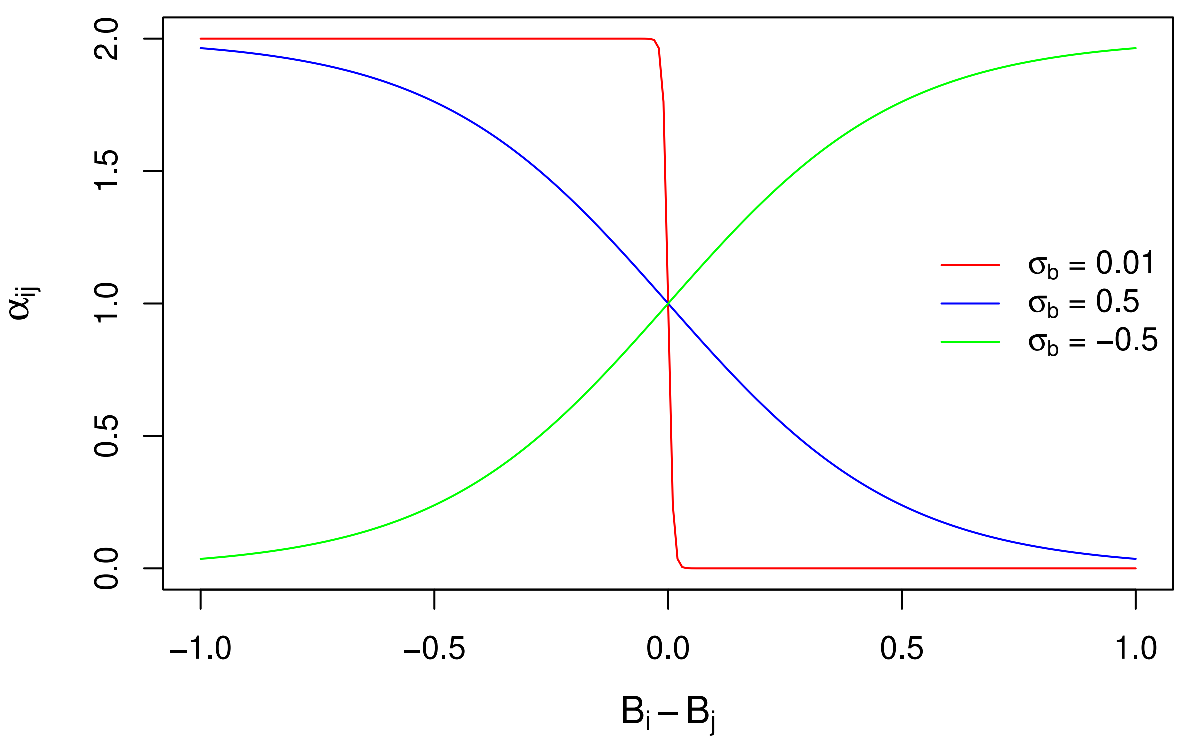

Figure 4: Shape of smooth function by Nattrass et al. with different values of σ_b (C=1)

However equation (6) is not defined when $sigma_b = 0$ and $B_j = B_i$, following suggestion of Nattrass et al. (2012) `asymmetric.competition.kernel` function set the strength of competition to 1 in this case.

3 `SeedProduction` function

This function calculates number of produced seeds for each individual. The number of seeds is random number from Bernoulli (zero or one seed) or Poisson distribution (unlimited number of seeds). The expected value of produced seeds (irrespectively to the distribution) for individual `i` in local community `k` depends on the competition for resources:

$$p_{ik} = b_0 * \max\left(\frac{K - \sum_{i\in k}\alpha_{ij}}{K}, 0\right) \tag{7}$$

Where: b_0 is the maximum probability of reproduction in competition free conditions; K is level of competition above which probability of reproduction becomes zero; α_{ij} = competitive effect of individual j on individual i, calculated from resource acquisition traits by the competition kernel functions.

References

Kisdi, É. (1999). Evolutionary Branching under Asymmetric Competition. *Journal of Theoretical Biology*, 197, 149–162.

MacArthur, R. & Levins, R. (1967). The Limiting Similarity, Convergence, and Divergence of Coexisting Species. *The American Naturalist*, 101, 377–385.

Nattrass, S., Baigent, S. & Murrell, D.J. (2012). Quantifying the Likelihood of Co-existence for Communities with Asymmetric Competition. *Bulletin of Mathematical Biology*, 74, 2315–2338.