Math 1906 Final Exam Solutions

Part	I	II	III	IV	Total
Score					

Part I: Fill in the blanks. (6 questions; 3 points for each; 18 points in total.)

Notice: you MUST write the answers in the following tables.

Number	1	2	3
Answer			
Number	4	5	6
Answer			

1. The first question $k>0$, text $f(x)=\ln x-\frac{x}{\mathrm{e}}+k$ text $(0,+\infty)$ text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text ____________2____________.

2. The second question $\vec{a}=(2,1,2)$, $\vec{b}=(4,-1,10)$, $\vec{c}=\vec{b}-\lambda\vec{a}$, text text text $\vec{a}\perp\vec{c}$, text text text text text text text text text text text text text text $\lambda=$ ______3______.

3. The third question $\begin{vmatrix} 1 & 2 \\ -3 & x \end{vmatrix}=0$, text text text text text text text text text text text text text text text text text text $x=$ ___________-6___________.

4. The fourth question $\alpha_1=(1,1,0),\alpha_2=(0,1,1),\alpha_3=(1,0,1)$, text $\beta=(4,5,3)$ text $\alpha_1,\alpha_2,\alpha_3$ text text text text text text text text text text text text text text text text text text text text text text text text text $\beta=$ ______$3\alpha_1+2\alpha_2+\alpha_3$______.

5. The fifth question ξ text text text $E\xi=3, D\xi=2$, text text text text text text text text text text text text text text text $E\xi^2=$ __________11__________.

6. The sixth question ξ text text text text η text text text text $\xi\sim N(1,4),\eta\sim N(2,5)$, text text text text text text text text text text text text text text text text text text text text text text text text text $\xi-2\eta\sim$ __________$N(-3,24)$__________.

Part II: Select one answer from four choices. (6 questions; 3 points for each; 18 points in total.)

Notice: you MUST write the answers in the following tables.

Number	1	2	3	4	5	6
Answer						

1. The first question text, text text text text text text text text text text text text text text text text text text text text text text text text ································· (C)

(A) first $\int f'(x)\,\mathrm{d}x = f(x)$ (B) second $\int \mathrm{d}f(x) = f(x)$

(C) third $\frac{\mathrm{d}}{\mathrm{d}x}\Big(\int f(x)\,\mathrm{d}x\Big) = f(x)$ (D) fourth $\mathrm{d}\Big(\int f(x)\,\mathrm{d}x\Big) = f(x)$

2. The second question $F(x)$ text $f(x)$ text, text ··························· (A)

(A) first choice $F(x)$ text $\Leftrightarrow f(x)$ text
(B) second choice $F(x)$ text text $\Leftrightarrow f(x)$ text
(C) third choice $F(x)$ text $\Leftrightarrow f(x)$ text text
(D) fourth choice $F(x)$ text text $\Leftrightarrow f(x)$ text text

3. The third question $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & x & 0 \\ 0 & 0 & 1 \end{pmatrix}$ text text $\lambda_1 = 1$ text $\lambda_2 = 2$, text $x =$ ······ (B)

(A) 2 (B) 1 (C) 0 (D) −1

4. The fourth question $f = 4x_1^2 - 2x_1x_2 + 6x_2^2$ text text text text text text text text text text text text text text text text text text text text text ························ (C)

(A) $\begin{pmatrix} 4 & -2 \\ -2 & 6 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix}$ (C) $\begin{pmatrix} 4 & -1 \\ -1 & 6 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$

5. The fifth question <u>wrong</u> text ···································· (B)

(A) first choice text text text text text text text text text
(B) second choice text text text text text text text text text
(C) third choice text text text text text text text text text
(D) fourth choice text text text text text text text text text

6. The sixth question X text $(X_1, \cdots, X_n)$ text <u>wrong</u> text text text text text text text text text text text text text text text text ································ (D)

(A) text text text (B) text n text

(C) $X_1, \cdots, X_n$ text (D) $X_1 = X_2 = \cdots = X_n$

Part III: Work out math questions. (6 questions; 8 points for each; 48 points in total.)

1. The first question $\int e^{2x}(\tan x+1)^2\,dx$.

Solution. $I=\int e^{2x}\sec^2 x\,dx+2\int e^{2x}\tan x\,dx$ ······2 points

$=\int e^{2x}\,d(\tan x)+2\int e^{2x}\tan x\,dx$ ······4 points

$=e^{2x}\tan x-2\int e^{2x}\tan x\,dx+2\int e^{2x}\tan x\,dx$ ······6 points

$=e^{2x}\tan x+C$ ······8 points

2. The second question $A(1,2,-1)$, $B(2,3,0)$, $C(3,3,2)$ text $\triangle ABC$ text text text text text text.

Solution. Text $\overrightarrow{AB}=(1,1,1),\overrightarrow{AC}=(2,1,3)$, ······2 points

text $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 1\\ 2 & 1 & 3\end{vmatrix}=(2,-1,-1)$, ······4 points

text $\triangle ABC$ text $S_{\triangle ABC}=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\sqrt{6}$. ······6 points

Text text $2(x-2)-(y-3)-z=0$, text $2x-y-z-1=0$. ······8 points

3. The third question $A=\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}$ text.

Solution. $A=\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}=\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 0 & -1 & -6 & 1 \\ 0 & -6 & -8 & 2 \end{vmatrix}=1\cdot(-1)^{2+1}\begin{vmatrix} 1 & 2 & 3 \\ -1 & -6 & 1 \\ -6 & -8 & 2 \end{vmatrix}$ ……4 points

$=-\begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & 4 \\ 0 & 4 & 20 \end{vmatrix}=-\begin{vmatrix} -4 & 4 \\ 4 & 20 \end{vmatrix}=-(-4\cdot 20-4\cdot 4)=96$ ……8 points

4. The fourth question, tex text $f=x_1^2+2x_1x_2-6x_1x_3+2x_2^2-12x_2x_3+9x_3^2$ text text $f=d_1y_1^2+d_2y_2^2+d_3y_3^2$.

Solution. $f=x_1^2+2x_1x_2-6x_1x_3+2x_2^2-12x_2x_3+9x_3^2$

$=x_1^2+2x_1(x_2-3x_3)+(x_2-3x_3)^2+x_2^2-6x_2x_3$

$=(x_1+x_2-3x_3)^2+x_2^2-6x_2x_3$ ……3 points

$=(x_1+x_2-3x_3)^2+x_2^2-2x_2\cdot 3x_3+(3x_3)^2-9x_3^2$

$=(x_1+x_2-3x_3)^2+(x_2-3x_3)^2-9x_3^2$ ……6 points

Text $y_1=x_1+x_2-3x_3$, $y_2=x_2-3x_3$, $y_3=x_3$,

text $f=y_1^2+y_2^2-9y_3^2$ text. ……8 points

5. The fifth question text text text 0.2 text text, text text 100 text text.
(1) text text text text text text ξ text 10 text 30 text.
(2) text text text text text ξ text 10 text 30 text.

Solution. $E\xi = np = 100 \cdot 0.2 = 20, D\xi = npq = 100 \cdot 0.2 \cdot 0.8 = 16.$ ······2 points

(1) $P(10 < \xi < 30) = P(|\xi - E\xi| < 10) \geqslant 1 - \frac{D\xi}{10^2} = 1 - \frac{16}{100} = 0.84.$ ······4 points

(2) $P(10 < \xi < 30) \approx \Phi_0\left(\frac{30-20}{\sqrt{16}}\right) - \Phi_0\left(\frac{10-20}{\sqrt{16}}\right)$ ······6 points

$= 2\Phi_0(2.5) - 1 = 2 \cdot 0.9938 - 1 = 0.9876$ ······8 points

6. The sixth question $N(\mu, \sigma^2)$ text text 16 text, text text text 3160, text text 100. Text text $H_0 : \mu = 3140$ text text $(\alpha = 0.01)$.

Solution. (1) Text text $H_0 : \mu = 3140$. ······2 points

(2) Text text text $T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1)$. ······3 points

(3) Text text $t_\alpha = t_\alpha(n-1) = t_{0.01}(15) = 2.947$. ······5 points

(4) Text text text $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3160 - 3140}{100/4} = 0.8$. ······7 points

(5) Text $|t| < t_\alpha$, text text H_0, text text text. ······8 points

Part IV: Work out math proofs. (2 questions; 16 points in total.)

1. (9 points) The first question $\{x_n\}$ text $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2+x_n}$. Text text text, text text text.

Proof. (1) Text, text $x_1 < 2$, text $x_k < 2$ text

$$x_{k+1} = \sqrt{2+x_k} < \sqrt{2+2} = 2,$$

Text text text text text n text $x_n < 2$, text text text. Text text

$$\frac{x_{n+1}}{x_n} = \sqrt{\frac{2}{x_n^2} + \frac{1}{x_n}} > \sqrt{\frac{2}{2^2} + \frac{1}{2}} = 1,$$

Text text text text text. Text text text text text, Text text text text. ······4 points

(2) Text text text text A, text text text text text text

$$A = \sqrt{2+A}.$$

Text text $A = 2$, text text $\{x_n\}$ text text text 2. ······8 points

2. (7 points) The second question A text B text, text A text $\bar{B}$ text.

Proof. $P(A \cdot \bar{B}) = P(A - B) = P(A - AB)$ ······2 points

$= P(A) - P(AB) = P(A) - P(A)P(B)$ ······4 points

$= P(A)(1 - P(B)) = P(A)P(\bar{B})$ ······6 points

Text text text A text text text $\bar{B}$ text text text. ······8 points

Appendix Some data may be used in the exam

$\Phi_0(0.5) = 0.6915$	$\Phi_0(1) = 0.8413$	$\Phi_0(2) = 0.9773$	$\Phi_0(2.5) = 0.9938$
$t_{0.01}(8) = 3.355$	$t_{0.01}(9) = 3.250$	$t_{0.01}(15) = 2.947$	$t_{0.01}(16) = 2.921$
$\chi^2_{0.005}(8) = 22.0$	$\chi^2_{0.005}(9) = 23.6$	$\chi^2_{0.005}(15) = 32.8$	$\chi^2_{0.005}(16) = 34.3$
$\chi^2_{0.995}(8) = 1.34$	$\chi^2_{0.995}(9) = 1.73$	$\chi^2_{0.995}(15) = 4.60$	$\chi^2_{0.995}(16) = 5.14$